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International Baccalaureate
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MATHEMATICS
STANDARD LEVEL
PAPER 1

Candidate session number

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Monday 11 November 2013 (afternoon)

Examination code

8	8	1	3	–	7	3	0	1

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

12 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 5]*

In the following diagram, $\overrightarrow{OP} = \boldsymbol{p}$, $\overrightarrow{OQ} = \boldsymbol{q}$ and $\overrightarrow{PT} = \frac{1}{2}\overrightarrow{PQ}$.

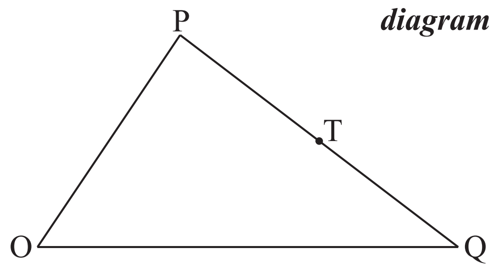


diagram not to scale

Express each of the following vectors in terms of $\boldsymbol{p}$ and $\boldsymbol{q}$,

(a) $\overrightarrow{QP}$; *[2]*

(b) $\overrightarrow{OT}$. *[3]*

2. *[Maximum mark: 6]*

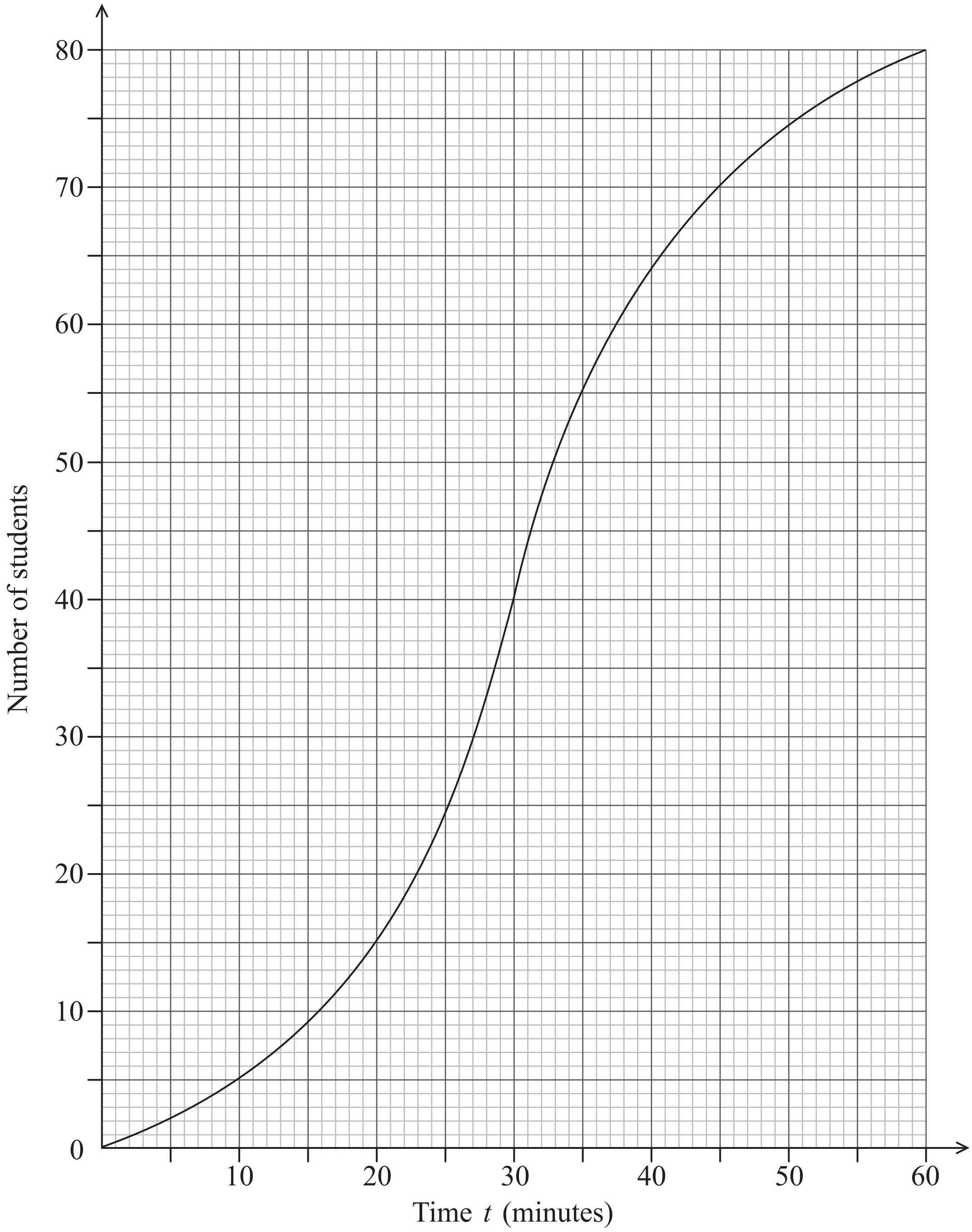
Let $\boldsymbol{A}=\begin{pmatrix}1 & 2\\ 3 & 0\end{pmatrix}$, $\boldsymbol{B}=\begin{pmatrix}2 & 1\\ 1 & 3\end{pmatrix}$ and $\boldsymbol{C}=\begin{pmatrix}2 & -4\\ 0 & 1\end{pmatrix}$.

(a) Find $\boldsymbol{AB}$. *[3]*

(b) Find $\det(\boldsymbol{AB}+\boldsymbol{C})$. *[3]*

3. *[Maximum mark: 7]*

The following is a cumulative frequency diagram for the time t, in minutes, taken by 80 students to complete a task.



(This question continues on the following page)

(Question 3 continued)

(a) Find the number of students who completed the task in less than 45 minutes. *[2]*

(b) Find the number of students who took between 35 and 45 minutes to complete the task. *[3]*

(c) Given that 50 students take less than k minutes to complete the task, find the value of k. *[2]*

4. *[Maximum mark: 6]*

Consider a function $f(x)$ such that $\int_1^6 f(x)\mathrm{d}x = 8$.

(a) Find $\int_1^6 2f(x)\mathrm{d}x$. *[2]*

(b) Find $\int_1^6 \left(f(x)+2\right)\mathrm{d}x$. *[4]*

5. *[Maximum mark: 7]*

Let $f(x) = \sin\left(x + \frac{\pi}{4}\right) + k$. The graph of f passes through the point $\left(\frac{\pi}{4}, 6\right)$.

(a) Find the value of k. *[3]*

(b) Find the minimum value of $f(x)$. *[2]*

Let $g(x) = \sin x$. The graph of g is translated to the graph of f by the vector $\begin{pmatrix} p \\ q \end{pmatrix}$.

(c) Write down the value of p and of q. *[2]*

6. *[Maximum mark: 6]*

Let $f(x) = \mathrm{e}^{2x}$. The line L is the tangent to the curve of f at $(1, \mathrm{e}^2)$.

Find the equation of L in the form $y = ax + b$.

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7. *[Maximum mark: 8]*

The equation $x^2 + (k+2)x + 2k = 0$ has two distinct real roots.

Find the possible values of k.

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 14]*

Let $f(x) = 3x - 2$ and $g(x) = \frac{5}{3x}$, for $x \neq 0$.

(a) Find $f^{-1}(x)$. *[2]*

(b) Show that $\left(g \circ f^{-1}\right)(x) = \frac{5}{x+2}$. *[2]*

Let $h(x) = \frac{5}{x+2}$, for $x \geq 0$. The graph of h has a horizontal asymptote at $y = 0$.

(c) (i) Find the y-intercept of the graph of h.

(ii) Hence, sketch the graph of h. *[5]*

(d) For the graph of h^{-1},

(i) write down the x-intercept;

(ii) write down the equation of the vertical asymptote. *[2]*

(e) Given that $h^{-1}(a) = 3$, find the value of a. *[3]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 16]*

The first three terms of a infinite geometric sequence are $m-1$, 6, $m+4$, where $m \in \mathbb{Z}$.

(a) (i) Write down an expression for the common ratio, r.

(ii) Hence, show that m satisfies the equation $m^2 + 3m - 40 = 0$. *[4]*

(b) (i) Find the two possible values of m.

(ii) Find the possible values of r. *[6]*

(c) The sequence has a finite sum.

(i) State which value of r leads to this sum **and** justify your answer.

(ii) Calculate the sum of the sequence. *[6]*

*Do **NOT** write solutions on this page.*

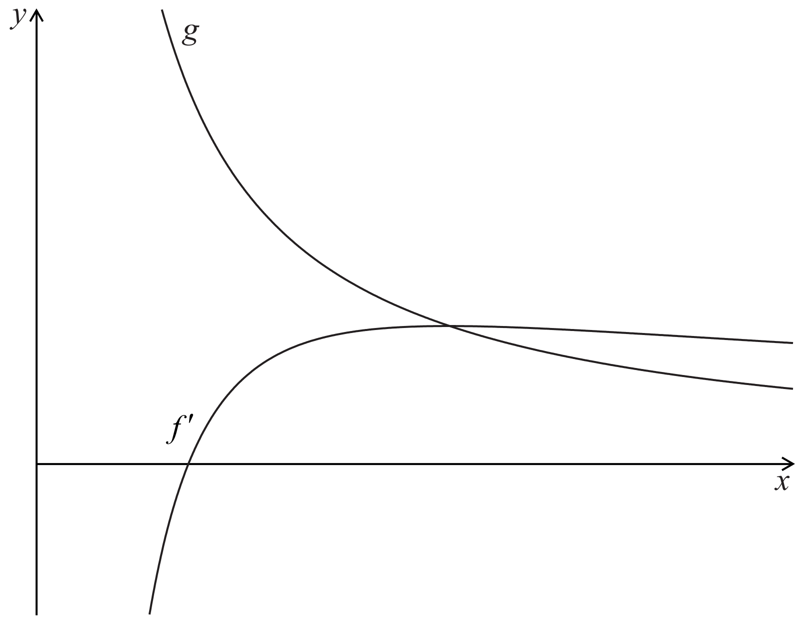
10. *[Maximum mark: 15]*

Let $f(x) = \frac{(\ln x)^2}{2}$, for $x > 0$.

(a) Show that $f'(x) = \frac{\ln x}{x}$. *[2]*

(b) There is a minimum on the graph of f. Find the x-coordinate of this minimum. *[3]*

Let $g(x) = \frac{1}{x}$. The following diagram shows parts of the graphs of f' and g.



The graph of f' has an x-intercept at $x = p$.

(c) Write down the value of p. *[2]*

The graph of g intersects the graph of f' when $x = q$.

(d) Find the value of q. *[3]*

(e) Let R be the region enclosed by the graph of f', the graph of g and the line $x = p$. Show that the area of R is $\frac{1}{2}$. *[5]*